THE POLARIZATION OF A LENS

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1. Introduction. In a previous paper [3], the author obtained inequalities comparing the capacity of a lens with various geometric quantities of a lens. A lens may be described simply as a solid of revolution determined by the intersection of two spheres. More precisely, if $c > 0$, the solid of revolution generated by revolving about the imaginary axis the area in the complex z-plane defined by the inequalities

$$\theta_1 \leq \arg \frac{z - c}{z + c} \leq \theta_2$$

is called a *lens*. We may suppose $0 < \theta_1 \leq \theta_2 < 2\pi$. It is, however, more convenient to characterize a lens in terms of its exterior angles. Accordingly we denote by α and β the exterior angles which the two portions of the boundary of the generating area make with the real axis. It is easily seen that $\beta = \theta_1$, $\alpha = 2\pi - \theta_2$. We shall assume, as we may without loss of generality, that $\alpha \leq \beta$. The sum of these angles, $\alpha + \beta$, is called the *dielectric angle* of the lens. Clearly we have $\alpha + \beta \leq 2\pi$, and hence we need consider only values of α not exceeding π. Sometimes it is convenient to introduce the radii a and b of the intersecting spheres; these are given by

$$c = a \sin \alpha = b \left|\sin \beta\right| .$$

It is clear that when $\alpha + \beta = \pi$ the lens becomes a sphere; and when $\alpha + \beta \geq \pi$, $\beta \leq \pi$, the lens is convex. When $\beta \neq 0$ and $\alpha \longrightarrow 0$, with a fixed, the lens becomes a sphere of radius a. When $\alpha, \beta \longrightarrow 0$ in such a manner that $\beta = k\alpha$, and a is kept fixed, the lens becomes two tangent spheres of radii a and a/k. When $\alpha, \beta \longrightarrow \pi$, with c fixed, the lens becomes a circular disk of radius c.

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