THE ASYMPTOTIC SOLUTIONS OF AN ORDINARY DIFFERENTIAL EQUATION IN WHICH THE COEFFICIENT OF THE PARAMETER IS SINGULAR

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1. Introduction. In this paper we are concerned with the solutions, for large values of the complex parameter λ, of the ordinary differential equation,

$$w''(s) - [\lambda^2 \sigma(s) + \tau(\lambda, s)] w(s) = 0 . \tag{1}$$

The variable s ranges over a region in the complex plane in which $\sigma(s)$ possesses a factor $(s - s_0)^{-2}$, where s_0 is some fixed point of the region. The asymptotic representations of the solutions of an equation formally identical with (1), but in which $\sigma(s)$ contains a factor $(s - s_0)^{\nu}$, $\nu > -2$, have been considered by Langer [3].

If equation (1) is considered over a region of the complex s-plane in which $\sigma(s)$ and $\tau(\lambda, s)$ are bounded, with $\sigma(s)$ bounded from zero, then it is possible to find a pair of asymptotic forms made up of elementary functions, each of these forms representing a solution over the entire region. If, however, $\sigma(s)$ becomes zero in the region under consideration, the asymptotic representations are complicated by the appearance of the Stokes' phenomenon. This necessitates abrupt but determinate changes in the asymptotic forms, if only elementary functions are used, as certain boundaries are crossed in the s- and λ-planes. The asymptotic representations of the solutions of (1) in this case have been considered by Langer [1] among others, and he has shown the Stokes' phenomenon to be quantitatively dependent upon the order of the zero of $\sigma(s)$. In a later paper [3], the theory was extended to include the cases where $\sigma(s)$ contains a factor $(s - s_0)^{\nu}$, $\nu > -2$, and $\tau(\lambda, s)$ has a pole of first or second order at s_0. He showed that the Stokes' phenomenon is engendered by and depends upon an infinity in either of the two coefficients in (1).

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