A PARTIAL DIFFERENTIAL EQUATION ARISING IN CONFORMAL MAPPING

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1. Introduction. During the past few years considerable attention has been given to the role played by kernel functions in conformal mapping, potential theory, and the theory of linear partial differential equations of elliptic type. The interest in this study has originated from the unifying influence which the concept of a kernel function introduces in these theories, and from the simple relationships discovered between the various kernels and the classical Green's and Neumann's functions [3-5, 8, 13]. Much of the older theory has been given a new interpretation, and a new light has been shed upon the study of canonical conformal maps, the Dirichlet problem, and the fundamental existence theorems [9, 10, 12]. The methods and results, which received their original impetus from investigations of functions of several complex variables [1], have been surprisingly simple and basic.

The present paper is devoted to the study of several kernel functions which arise in conformal mapping and in mathematical physics, and to the investigation of some eigenvalue problems related to these kernels. We show that the kernel function associated with the norm

$$\iint_D |\phi(z)|^2 \rho(z)\, dxdy$$

of analytic functions $\phi(z)$ of the complex variable z in a plane domain D can be expressed in terms of the Green's function of the partial differential equation

$$\frac{\partial}{\partial \bar{z}} \frac{1}{\rho(z)} \frac{\partial}{\partial z} S(z) = 0,$$

where

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right), \quad \frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right), \quad z = x + iy.$$

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