SOME INEQUALITIES IN CERTAIN NONORIENTABLE RIEMANNIAN MANIFOLDS

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1. Introduction. As is well known, the projective plane and the Moebius strip are nonorientable manifolds of dimension two. After introducing a Riemannian metric on each of them, we obtain two 2-dimensional nonorientable Riemannian manifolds. For convenience of reference, let us denote them by M^2_{p2} and M^2_m, respectively. Each of these manifolds has an area A. Moreover, there exists a family of closed curves, which are not homotopic to zero, on each manifold; and hence the set of the lengths of all these closed curves in consideration has a positive greatest lower bound, a. The purpose of this paper is to investigate the relationship between these two geometrical constants, A and a. It is found that, in each case, there exists an inequality [1] connecting them, of the form

$$A \geq ka^2, \tag{1}$$

k being a constant depending only on the conformal character of the Riemannian manifold. To establish such inequalities and to determine the corresponding best possible constants are the two central problems in this investigation.

For the time being, the projective plane is used in the following realization: it is given as the unit sphere with identification of diametrically opposite points. We assume further that the metric on M^2_{p2} is given by

$$ds^2 = g(p)\ d\rho^2,$$

$d\rho^2$ being the line element of the unit sphere taken from the embedding Euclidean space; $g(p) \in C_\omega$, $g(p) > 0$ for any point p on the manifold. As for the Moebius strip, we assume that it is given by the strip

$$-\beta < y < \beta,$$

with identification given by the fundamental group

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