DAVIS'S CANONICAL PENCILS OF LINES

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1. **Introduction.** The purpose of the present paper is to contribute to the study of two pencils of lines which appear in the development of the theory of conjugate nets on an analytic nonruled surface in ordinary space. The surface is herein referred to its asymptotic curves as parametric.

In his Chicago doctoral dissertation, *Contributions to the theory of conjugate nets*, W. M. Davis defined and studied several canonical configurations, considering the conjugate net as parametric. Among these configurations there are two pencils of lines, called Davis's first and second canonical pencils, which are studied in this paper.

Investigation is made of certain polar relations of the lines of the two pencils with respect to the bundle of quadrics each of which has contact of at least the third order with both curves of a conjugate net at a point. Certain loci which arise in the relation of the two pencils of lines to a pencil of conjugate nets at a point on a surface are studied. A generalization of Davis's canonical quadric is made, and some properties of the resulting one-parameter family of quadrics are demonstrated. Theorems relating to all the lines of one or both of the canonical pencils are shown to include theorems previously proved about certain particular lines.

2. **Analytic basis.** Let the projective homogeneous coordinates x_1, x_2, x_3, x_4 of a point P_x [3, pp. 89, 105-113, 115-120, 180-190], on an analytic nonruled surface S in ordinary space be given by the parametric vector equation $x = x(u, v)$. Suppose the surface to be referred to its asymptotic net. Then P_x satisfies two partial differential equations of the form

$$(2.1)\qquad \begin{aligned} x_{uu} &= px + \theta_u x_u + \beta x_v, \\ x_{vv} &= qx + \gamma x_u + \theta_v x_v \qquad (\theta = \log \beta\gamma). \end{aligned}$$

Let x_1, x_2, x_3, x_4, the local point coordinates of P_x, be referred to the tetrahedron of reference whose vertices are the points x, x_u, x_v, x_{uv}.

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