UNIFORMLY ACCESSIBLE JORDAN CURVES THROUGH LARGE SETS OF RELATIVE HARMONIC MEASURE ZERO

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1. **Introduction.** Let R be a Jordan region in the w-plane, E a set of points on the boundary C of R, and $w = f(z)$ a continuous schlicht mapping of the closed unit disc in the z-plane upon the closure of R, conformal in the open unit disc. The set E is said to have harmonic measure zero relative to R provided its image $f^{-1}(E)$ on the unit circle is a set of Lebesgue measure zero.

Lohwater and Seidel [4] have constructed a Jordan region R whose boundary passes through a linear set E of positive measure in such a way that E has harmonic measure zero relative to R. Also, Lohwater and Piranian [3] have described a Jordan region R whose boundary passes through a set E of positive two-dimensional Lebesgue measure in such a way that E has harmonic measure zero relative to R. In both cases, the set E consists of points each of which is not the end point of any rectifiable arc whose remaining points lie in R. And in both cases the fact that the points of E are not accessible by rectifiable paths in R serves no other purpose than to permit the application of Lavrentiev's theorem on finite accessibility [1] (see also Tsuji [9, p. 99] and Seidel and Walsh [8, p. 143]) in proving that the set E has harmonic measure zero relative to R.

The author of the present note holds the view that bizarre examples are acceptable only as long as no simple replacements are available; and he believes that the geometrically important aspect of the earlier constructions is not the fact that the points of E are not finitely accessible from R, but merely this, that the points of E cannot be reached from the interior of R except along paths through narrow corridors (whose tortuous character is irrelevant, except in the proofs). The motivation of this belief is evident from the proof that will be given of the following proposition.

THEOREM 1. *There exists a Jordan region R whose boundary C passes through a linear set of positive Lebesgue measure and of harmonic measure zero relative to R, and which has the property that each point of C can be reached from a certain fixed point in R by an arc whose length is less than one and whose interior points all lie in R.*

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