THE BOUNDEDNESS OF THE SOLUTIONS OF A DIFFERENTIAL EQUATION IN THE COMPLEX DOMAIN

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1. **Introduction.** Let $Q(z)$ be an analytic function of the complex variable z in a domain. In the following we shall be concerned with the differential equation

$$\frac{d^2W}{dz^2} + Q(z)\, W = 0. \tag{1}$$

Only those solutions $W(z)$ of (1) which are distinct from the trivial solution $(\equiv 0)$ shall be considered.

For a real-valued continuous solution $y(x) \not\equiv 0$ of the differential equation

$$\frac{d^2y}{dx^2} + f(x)\, y = 0, \tag{2}$$

where $f(x)$ is a real-valued piecewise continuous function of the real variable x for $0 \leq x < \infty$, N. Levinson [1] has shown that the rapidity with which $y(x)$ can grow, and the rapidity with which it can tend to zero, both depend on the growth of $\alpha(x)$, where

$$\alpha(x) = \int_0^x |f(x) - a|\, dx, \tag{3}$$

and a is a real positive constant. More precisely, he showed that

$$y(x) = O\left(\exp\left[\frac{1}{2}\, a^{-1/2}\, \alpha(x)\right]\right), \tag{4}$$

and that if $\alpha(x) = O(x)$ as $x \longrightarrow \infty$, then

$$\limsup_{x \to \infty} |y(x)| \exp\left[\frac{1}{2}\, a^{-1/2}\, \alpha(x)\right] > 0. \tag{5}$$

If there exists a positive constant a such that $\alpha(x)$ converges as $x \longrightarrow \infty$, then

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