EVALUATION OF AN INTEGRAL OCCURRING IN SERVOMECHANISM THEORY

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1. Introduction. In the study of dynamical systems in general, and servomechanisms in particular, it is often required to determine the (constant) coefficients in a linear, ordinary, differential equation in such a way as to minimize an integral involving the square of the difference between the solution of the equation and a known function. The latter may be given in either analytical or numerical form. In the design of a servomechanism the known function is the "input"; the solution of the equation is the "output"; and the coefficients of the equation are the circuit constants to be determined. A similar problem arises in the study of aircraft flight records, in which the known function is any of the dynamic variables used to describe the motion, and the coefficients are the so-called aerodynamic derivatives, the determination of which is the purpose of the flight.

Mathematically similar problems also arise in the analysis of a mixture of radioactive substances or of bacteria. The known function is, say, the total weight of the mixture as a function of time, and the unknown coefficients are the relative weights of the different substances initially present.

All such problems can be solved by the method of least squares, and the procedure always leads, at a certain stage, to the evaluation of an integral of a particular type. This integral has been studied by R. S. Phillips [3, Chap. 7, §7.9], who has given a procedure for its evaluation and a short table of results. The purpose of the present note is to derive a simple, explicit formula for this integral.

2, Evaluation of the integral. The integral to be evaluated is

(1) $$I = \frac{1}{2\pi i}\int_{-\infty i}^{\infty i} \frac{g(x)}{h(x)\,h(-x)}\,dx,$$

where

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