SPECTRAL THEORY II. RESOLUTIONS OF THE IDENTITY

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Introduction

In attempting to extend elementary divisor theory to the case of a linear operator on a complex Banach space one is naturally led to a consideration of the various equivalent definitions of the multiplicity $\nu(\lambda)$ of a complex number λ as a root of the minimal equation of a finite matrix T. Of the numerous equivalent definitions of this integer we have found only one which seems to have some virtue when applied to the infinite dimensional case. That one is as follows: $\nu(\lambda)$ is the smallest positive integer or zero for which

$$|\xi - \lambda|^{\nu(\lambda)} \; |(\xi - T)^{-1}|$$

is bounded for ξ near λ. Thus the rate of growth of the resolvent

$$T(\xi) = (\xi - T)^{-1}$$

for ξ near λ determines $\nu(\lambda)$. In this paper we consider the problem of determining conditions on the rate of growth and the mean rate of growth of the resolvent which are necessary and sufficient for a complete reduction of a linear operator on a complex Banach space. What is to be meant by a "complete" reduction? There are several apparent meanings that might be given to the notion of the resolution of the identity for an operator, all reducing to the classical one in the case of a finite matrix. For example, are we to require that E_σ be defined for all Borel sets σ or for σ in some sufficiently large subalgebra; should it be countably or just finitely additive; should it be bounded or not? All problems are legitimate and in this paper we have chosen the most restrictive of all the obvious interpretations. Consequently the conditions found on $T(\xi)$ are restrictive and the corresponding class of operators is small. On the other hand, such operators have many important properties not shared by operators outside this class. Other meanings for the notion of resolution of the identity will be

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