ON SELF-ADJOINT DIFFERENTIAL EQUATIONS OF SECOND ORDER

RUTH LIND POTTER

Introduction. This paper is concerned with the behavior near $x = \infty$ of solutions of the self-adjoint differential equation

$$[r(x)y']' + p(x)y = 0, \tag{1}$$

where $r(x) > 0$ and $r(x)$ and $p(x)$ are continuous for positive values of x. A solution is said to oscillate near $x = \infty$ if it has no largest zero. We study the oscillation and boundedness of solutions of equations of the form (1). Repeated use is made throughout the paper of the Sturm comparison and separation theorems and of two theorems due to Leighton [6; 5]. Leighton's theorems are the following.

THEOREM L_1. *If $r(x)$ and $p(x)$ are continuous and $r(x) > 0$ on the interval $0 < x < \infty$, and*

$$\lim_{x\to\infty} \int_1^{\infty} \frac{dx}{r(x)} = \infty \quad \textit{and} \quad \lim_{x\to\infty} \int_1^{x} p(x)dx = \infty,$$

then every solution of (1) vanishes infinitely often on the interval $(1, \infty)$.

THEOREM L_2. *If $r(x)$ and $p(x)$ are continuous, and $r(x)p(x)$ is a positive monotone function of x for x large, a necessary condition that solutions of (1) be oscillatory near $x = \infty$ is that not both limits*

$$\lim_{x\to\infty} \int_1^{x} \frac{dx}{r(x)}, \qquad \lim_{x\to\infty} \int_1^{x} p(x)dx$$

exist and are finite.

We proceed to the study of conditions under which solutions of equation (1)

Received May 5, 1952. The author is indebted to Professor Walter Leighton for helpful suggestions in the preparation of this paper. Part of the work was done while the author was employed under contract N9onr-95100 with the Office of Naval Research.

Pacific J. Math. 3 (1953), 467-491