THE SPHERICAL CURVATURE OF A HYPERSURFACE IN EUCLIDEAN SPACE

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1. Introduction. Let V_n be a hypersurface immersed in a Euclidean space S_{n+1}. Let P be a point of V_n corresponding to the point P' of the hyperspherical representation G_n of V_n. Let V denote the extension of a region ϕ of V_n, and V' the extension of the corresponding hyperspherical region ϕ' of G_n. If the region around P tends to zero, the ratio V'/V tends to a limit Γ, which is called the *spherical curvature* of V_n at P [1, pp. 258-261]. It is found that $\Gamma = |\Omega/g|$, where $g = |g_{ij}|$ and $\Omega = |\Omega_{ij}|$ are respectively the determinants of the coefficients of the first and the second fundamental forms of V_n. In this note, some properties of the spherical curvature are studied, and new interpretations of the Gaussian curvature are derived.

The notation of Eisenhart [2] will be used for the most part.

2. Some properties. Let a real and analytic hypersurface V_n be defined by

$$y^\alpha = y^\alpha(x^1, \cdots, x^n) \qquad (\alpha = 1, \cdots, n+1),$$

referred to a Cartesian coordinate system y^α in a Euclidean space S_{n+1}. Let a vector-field v in V_n be defined by

$$v^\alpha = p^i \partial y^\alpha / \partial x^i \qquad (i = 1, \cdots, n),$$

where the v^α are real and analytic functions of the x^i. Let C be a curve of V_n. The normal curvature vector of v with respect to C at P is defined as the normal component of the derived vector of the vector-field v along C at P [3]. Let κ denote a nonzero extreme value of the magnitudes of the normal curvature vectors of v with respect to all curves of V_n at P. Then κ, which is called a principal curvature of v at P, is defined by

$$(2.1) \qquad |\Psi_{ij} - \kappa^2 g_{ij}| = 0,$$

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