# REMARK ON THE PRECEDING PAPER OF CHARLES LOEWNER

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**1. Introduction.** In the preceding paper, Charles Loewner constructed certain Jordan curves with the property that the clamped plates bounded by such Jordan curves have an oscillating Green's function. The question concerning the sign of the Green's function has been raised by J. Hadamard, and this problem has been pursued recently by R. J. Duffin and P. R. Garabedian. The construction of Loewner is based on a method due to N. Muskhelichvili using appropriate conformal mappings.[1]

The purpose of the present note is to construct such Jordan curves in an elementary manner. For the sake of completeness we repeat a few definitions to be found in the preceding paper.

A function of $u(x, y)$ defined in a domain $g$ and having therein continuous partial derivatives of the fourth order is called a biharmonic function in $g$ if it satisfies the biharmonic equation

$$(1) \qquad \nabla^4 u = \nabla^2 \nabla^2 u = \frac{\partial^4 u}{\partial x^4} + 2 \frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} = 0.$$

Let $g$ be a connected domain bounded by a finite number of analytic arcs. Let $q$ be a fixed point in $g$. The Green's function $\Gamma(p) = \Gamma(p; q)$ of $g$ with respect to $q$ is a function of the variable point $p = p(x, y)$ satisfying the following conditions:

(a) $\Gamma$ is a biharmonic function of $p$ except at the singular point $q$. Denoting by $r$ the distance of $p$ from $q$, we have

$$(2) \qquad \Gamma = r^2 \log r + k,$$

where $k(p) = k(x, y)$ is biharmonic in $g$ without exception.

(b) On the boundary of $g$ we have the conditions:

[1] See the References given in the paper of Loewner.

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