# SOME HAUSDORFF MEANS WHICH EXHIBIT THE GIBBS' PHENOMENON

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**1. Introduction.** The regular Hausdorff mean of order $n$ with kernel $g(x)$ for the sequence $(s_k)$ is defined by

$$h_n = h_{n,\,g} = \sum_{k=0}^{n} \binom{n}{k} s_k \int_0^1 t^k (1-t)^{n-k}\, dg(t),$$

where $g(x)$ is of bounded variation on the interval $0 \leq x \leq 1$, $g(1) - g(0) = 1$, and $g(0+) = g(0)$. The integral in the definition being a Stieltjes integral, it is clear that $g(0)$ may be taken to be zero.

For the sequence

$$s_n(x) = \sum_{k=1}^{n} \frac{\sin kx}{k},$$

Otto Szász [3] has proved the following result: If, as $n \longrightarrow \infty$, $x_n \longrightarrow 0+$ and $nx_n \longrightarrow A \leq \infty$, then

$$h_{n,g}(x_n) \longrightarrow \int_0^1 \operatorname{Si}(Ax)\, dg(x),$$

where

$$\operatorname{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt.$$

He defines the Gibbs' ratio for the kernel $g(x)$ to be

$$F(g) = \max_{A>0} \frac{2}{\pi} \int_0^1 \operatorname{Si}(Ax)\, dg(x).$$

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