ON THE HITCHCOCK DISTRIBUTION PROBLEM

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1. Introduction. Frank L. Hitchcock [1] has offered a mathematical formulation of the problem of determining the most economical manner of distribution of a product from several sources of supply to numerous localities of use, and has suggested a computational procedure for obtaining a solution of his system in any particular case. L. Kantorovitch [2], Tjalling C. Koopmans [3], George B. Dantzig [4b], C. B. Tompkins [5], Julia Robinson [7; 8], Alex Orden [6], and others [4] have also discussed the computational aspects of this problem; paper [5] illustrates the use of the "projection method," due to C. B. Tompkins, as a computational process applicable to either of the Fundamental Problems of the present paper.

We shall be concerned only with the mathematical justification of computational procedure, and shall limit our attention to one specific method of solution of general validity. No attempt will be made to compare the various methods already proposed, either as to their mathematical similarity or as to their relative efficiency in any particular case.

2. The problem. The problem is to find a set of values of the mn variables x_{ij}, subject to the following conditions:

$$\sum_{i=1}^{m} x_{ij} = c_j, \quad \sum_{j=1}^{n} x_{ij} = r_i, \tag{2.1}$$

$$x_{ij} \geq 0, \tag{2.2}$$

Received January 25, 1952. The author's interest in the problem was aroused by papers on transportation theory presented by Koopmans [4a] and Dantzig [4b] at a conference on linear programming in Chicago during June, 1949, under the auspices of the Cowles Commission for Research in Economics of the University of Chicago. Several other papers presented at this conference are of closely related interest. Professor Koopmans, in his Introduction to the Conference Proceedings [4], also discussed the background and interrelationship of the conference papers—including the bearing of some of these on the Hitchcock distribution problem. The results of the present paper have been presented in three seminar lectures: once in December, 1949, at The RAND Corporation in Santa Monica, once in July, 1950, at the Institute for Numerical Analysis of the National Bureau of Standards in Los Angeles, and once in June, 1951, at the National Bureau of Standards in Washington, D.C. The author is especially indebted to Dr. D.R. Fulkerson, who has given real assistance in simplifying notation and proofs of theorems, for a careful reading of the manuscript.

Pacific J. Math. 3 (1953), 369-386