SUBFUNCTIONS OF SEVERAL VARIABLES

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1. Introduction. Convex functions have been generalized in the following two directions: to subharmonic functions [5] of two (or more) independent variables, by replacing the dominating family $\{F(x)\}$ of linear functions, or solutions of the differential equation

$$\frac{d^2 F}{dx^2} = 0,$$

with a family of harmonic functions $\{F(x, y)\}$, or solutions of the partial differential equation

$$\Delta F \equiv \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = 0; \tag{1}$$

and to subfunctions [1] of one variable, by replacing the dominating family of linear functions with a more general family of functions of one variable having certain geometric features in common with the family of linear functions.

We shall here combine the foregoing considerations, generalizing subharmonic functions by replacing the dominating family of harmonic functions with a more general family of functions of two (or more) independent variables.

Bonsall [2] has recently considered some properties of subfunctions of two independent variables relative to the family of solutions of the second-order elliptic partial differential equation

$$\Delta F + a(x, y)\frac{\partial F}{\partial x} + b(x, y)\frac{\partial F}{\partial y} + c(x, y)F = 0.$$

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