AN OPERATIONAL CALCULUS FOR OPERATORS WITH SPECTRUM IN A STRIP

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1. Introduction. Let X be a complex Banach space, and T be a closed distributive operator whose domain and range are in X. We suppose the spectrum $\sigma(T)$ of T does not cover the whole plane, and write

$$(\lambda I - T)^{-1} = R_\lambda(T)$$

for $\lambda \notin \sigma(T)$. In the case that T is bounded, N. Dunford [2] and A. E. Taylor [13] have defined an operational calculus for T by the formula

$$f(T) = \frac{1}{2\pi i}\int_C f(\lambda)\, R_\lambda(T)\, d\lambda, \tag{1.1}$$

where f is analytic on $\sigma(T)$, and C is a suitable bounded contour enclosing $\sigma(T)$. Such functions f form an algebra, and the mapping $f \longrightarrow f(T)$ is a homomorphism of this algebra into the algebra of bounded operators on X.

When T is assumed to be closed but not bounded, the problem of developing an operational calculus for T meets with the difficulties that the domain $D(T)$ is a proper subspace, and $\sigma(T)$ is in general unbounded. A modification of (1.1),

$$f(T) = f(\infty)I + \frac{1}{2\pi i}\int_C f(\lambda)\, R_\lambda(T)\, d\lambda, \tag{1.2}$$

has been used by Taylor [14] when f is analytic on $\sigma(T)$ and at infinity. Here C is a bounded contour enclosing the singularities of f. Although most of the theory for the bounded case may be carried over, the class of functions f is restricted; and polynomials in T, being unbounded operators, need a separate

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