PLANE GEOMETRIES FROM CONVEX PLATES

MARLOW SHOLANDER

1. Introduction. It is shown below that to each member of a general class of two-dimensional convex bodies there corresponds an affine geometry in the sense of Artin [1] and an S. L. space in the sense of Busemann [4].

A two-dimensional convex body is called a *convex plate.* For the few elementary properties of such plates assumed here, see [3].

Let K be a convex plate, and let K^0 denote its boundary curve. All constructions are to be made in the plane E of K. Consider an arbitrary direction ϕ in E and the two lines of support to K in this direction. Let t_0 be the line of support whose associated half-plane in the direction $\phi + \pi/2$ contains K. Let t_1 be the other line of support. For $0 < i < 1$, let t_i be the line parallel to t_0 which divides line segments extending from t_0 to t_1 in the ratio of i to $1 - i$. Let t_i cut K^0 at points R_i and T_i so that the directed segment $R_i T_i$ has direction ϕ.

For $0 < i < 1$ and $0 < j < 1$, let S_{ij} be the point which divides $R_i T_i$ in the ratio of j to $1 - j$. The set $s_j = \bigcup_i S_{ij}$ is an open Jordan arc whose endpoints are points of contact of t_0 and t_1 with K. A set s_j is called a *strut.* Other struts may be obtained by varying ϕ. When the direction needs emphasis, the above notations are modified by affixing the angle in parentheses, for example, $R_i(\phi)$ or $s_j(\phi)$. Two struts with no common points or all points in common are called *parallel.* Clearly $s_j(\phi)$ and $s_k(\phi)$ are parallel.

Under the name *Durchlinien,* Zindler [6] studied struts of the form $s_{1/2}(\phi)$. It is easy to see that $s_{1/2}(\phi)$ halves the area of K, and that the centroid of K is contained in the convex hull of this strut.

2. A preliminary theorem. This section is devoted to a proof of the following theorem. An edge of K is defined as a (*maximal*) *line segment in* K^0.

THEOREM. *If for distinct directions ϕ and ψ, struts $s_i(\phi)$ and $s_j(\psi)$ meet at distinct points P and Q, they meet at all points of the segment PQ. Such segments of intersection occur if and only if K has at least two edges.*

Received May 19, 1952. A part of this paper was written while the author was under contract to the Office of Naval Research.

Pacific J. Math. 3 (1953), 667-671