ALTERNATING METHOD ON ARBITRARY RIEMANN SURFACES

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1. Introduction. Schwarz gave the first rigorous construction of harmonic functions with given singularities on closed Riemann surfaces, by means of his alternating method for domains with annular intersection [16]. The method also is directly applicable to open Riemann surfaces of finite genus, since these can always be continued so as to form closed surfaces [7; 8]. For surfaces of infinite genus, this continuation is no longer possible. But if the surface is of parabolic type, Schwarz's method can still be used, a "null boundary" having no effect on the behaviour of the alternating functions [5; 11]. In the general case, there are two obstacles which prevent using Schwarz's method as such. First, if the surface has a large (ideal) boundary, the alternating functions are not determined by their values on the relative boundaries. Second, Schwarz's convergence proof fails, since the Poisson integral is inapplicable on arbitrary Riemann domains. We are going to show that, by certain changes of Schwarz's original method, these difficulties can be overcome.

This paper is a detailed exposition of a reasoning outlined in preliminary notes [9-11]. The manuscript of the paper was communicated (in French) to the Helsinki University in December, 1949. In the meanwhile, the author published a linear operator method [13], which also can be used to establish the results of these notes. A presentation of the classical alternating method for arbitrary Riemann surfaces seems, however, to have independent interest from a methodological viewpoint; such a presentation is the purpose of this paper.

The alternating method on Riemann surfaces, as sketched in [9-11], was referred to also in the recent papers of Kuramochi [1], Kuroda [2], Mori [3], and Ohtsuka [6]. A historical note on the method was given in [15].

2. Functions with vanishing conjugate a_0-periods. We start with two lemmas, which are basic for the alternating procedure.

Let R be an arbitrary Riemann surface, and G a subdomain, compact or not. The relative boundary a_0 of G, that is, the set of boundary points of G, interior

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