THE SPACE H^p, $0 < p < 1$, IS NOT NORMABLE

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1. Introduction. For $p > 0$, the space H^p is defined to be the class of functions $x(z)$ of the complex variable z, which are analytic in the interior of the unit circle, and satisfy

$$\sup_{0 \le r < 1} \int_0^{2\pi} |x(re^{i\theta})|^p \, d\theta < \infty.$$

Set

$$A_p(r; x) = \left(\frac{1}{2\pi} \int_0^{2\pi} |x(re^{i\theta})|^p \, d\theta \right)^{1/p}$$

and

$$||x|| = \sup_{0 \le r < 1} A_p(r; x).$$

S. S. Walters has shown [2] that H^p, $0 < p < 1$, is a linear topological space under the topology: $U \subset H^p$ is open if $x_0 \in U$ implies the existence of a "sphere" S: $||x - x_0|| < r$ such that $S \subset U$. He conjectured in [3] that H^p, $0 < p < 1$, does not have an equivalent normed topology, and it is shown here that this conjecture is correct. Since the conjugate space $(H^p)^*$ has sufficiently many members to distinguish elements of H^p, the space H^p, $0 < p < 1$, affords an interesting nontrivial example of a locally bounded linear topological space which is not locally convex.

2. *Proof.* For $x \in H^p$, $p > 0$, it is known [4, 160] that $A_p(r; x)$ is a nondecreasing function of r. Consequently, if $P(z)$ is a polynomial, then $P \in H^p$ and $||P|| = A_p(1; P)$. This observation will be used below.

According to a theorem of Kolmogoroff [1], a linear topological space has an equivalent normed topology if and only if the space contains a bounded open convex set. It will be shown here that the "sphere" K_1: $||x|| < 1$ of H^p,

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