DISTRIBUTION OF ROUND-OFF ERRORS FOR RUNNING AVERAGES

R. E. GREENWOOD AND A. M. GLEASON

1. Statement of the problem. Let $G_1, G_2, \cdots$ be scores (positive integers) obtained in a sequence of plays in a certain game. For purposes of handicapping matches it is desired to use running averages, and on the hypothesis that the score of the last play is more significant than any prior score, the following formula is used for computing the running averages $\{S_n\}$:

$$S_{n+1} = \frac{(k-1)\,S_n + G_{n+1}}{k} \tag{1.1}$$

where k is a positive integer. Certain modifications in (1.1) may be necessary when $n < k$.

The running averages defined by (1.1) are not necessarily integers. It is therefore convenient to define a rounded running average (which will be integral) by the relation

$$T_{n+1} = \frac{(k-1)\,T_n + G_{n+1} + D}{k}. \tag{1.2}$$

It is convenient to use three set of values for D in the foregoing relation.

Case A. For k odd, $D = A \in \left\{\frac{-k+1}{2}, \frac{-k+3}{2}, \cdots, \frac{k-1}{2}\right\}$.

Case B. For k even, $D = B \in \left\{\frac{-k}{2} + 1, \frac{-k}{2} + 2, \cdots, \frac{k}{2}\right\}$.

Case C. For k even, $D = C \in \left\{\frac{-k}{2}, \frac{-k}{2} + 1, \cdots, \frac{k}{2}\right\}$.

For each $n \geq k$ define the error E_n by the relation

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