A METHOD OF GENERAL LINEAR FRAMES IN RIEMANIAN GEOMETRY, I

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1. Introduction. In this paper we shall derive the basic quantities of Riemannian geometry, such as parallelism, curvature tensors, and so on, from a consideration of all linear frames in the various tangent spaces. This procedure has the advantage of subsuming both the classical approach through local coordinate frames and the more modern approach through orthonormal frames. The exact connection between these methods is thus made quite explicit.

The principal machinery used here is the exterior differential calculus of É. Cartan. (See [**1**, p. 201-208; **2**, p. 33-44; **3**, p. 4-6; **4**, p. 146-152; **7**, p. 3-10].) We shall follow the notation of Chern [**3**] with exceptions that we shall note in the course of the paper. It is important to keep in mind the following specific points of this calculus.

On a differentiable manifold of dimension n one has associated with each $p = 0, 1, 2, \cdots$ the linear space of exterior differential forms of degree p (p-forms). The coefficients form the ring of differentiable functions on the manifold. The 0-forms are simply the functions themselves, and the only p-form with $p > n$ is the form 0. Locally, if $u^1, \cdots, u^n$ is a local coordinate system then a one-form ω may be written

$$\omega = \sum f_i(u)\, du^i; \tag{1.1}$$

and, more generally, a p-form ω may be written

$$\omega = \sum_{(1 \le i_1 < \cdots < i_p \le n)} f_{i_1 \cdots i_p}(u)\, du^{i_1} \cdots du^{i_p} \tag{1.2}$$

$$= \frac{1}{p!} \sum f_{i_1 \cdots i_p}(u)\, du^{i_1} \cdots du^{i_p} \quad \text{with the } f_{(i)} \text{ skew-symmetric.}$$

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