ON THE COMPLEX ZEROS OF FUNCTIONS OF STURM-LIOUVILLE TYPE

CHOY-TAK TAAM

1. Let $Q(z)$ **be an analytic function** of the complex variable z in a region D. In the present paper only those solutions of

$$W'' + Q(z)W = 0 \tag{1.1}$$

which are distinct from the trivial solution ($\equiv 0$) shall be considered.

In this paper the following results shall be established.

THEOREM 1. *Suppose that the following conditions are satisfied:*

(a) *the circle* $|z| \leq R$ *is contained in* D,

(b) $W(z)$ *is a solution of* (1.1), $W(0) \neq 0$,

(c) $n(r)$ *is the number of zeros of* $W(z)$ *in* $|z| \leq r$, $r < R$.

Then $n(r)$ *satisfies the inequality*

$$n(r) \leq (\log (Rr^{-1}))^{-1}[\log (1 + R|W'(0)|\,|W(0)|^{-1}) + (2\pi)^{-1} \int_0^{2\pi}\int_0^R (R-t)\,|Q(t\,e^{i\theta})|\,dtd\theta]. \tag{1.2}$$

COROLLARY 1.1. *Suppose that the following conditions are satisfied:*

(a) $Q(z)$ *is a polynomial of degree* k,

(b) *conditions* (b) *and* (c) *of Theorem* 1 *hold.*

Then $W(z)$ *is an integral function of order at most* $k+2$. *Furthermore, as* $r \longrightarrow \infty$,

$$n(r) = O(r^{k+2}). \tag{1.3}$$

Obviously the result of Theorem 1 is not good if r is close to R. Also it

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