A NOTE ON THE HÖLDER MEAN

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1. Introduction. Of the two better-known generalizations of the simple arithmetic mean, the Hölder mean and the Cesàro mean, the latter has been the more extensively studied. This is primarily due to the equivalence of the two when used to define summability methods and to the following formulas. If we define C_n^k, the k^{th} order Cesàro mean of the terms $S_0, S_1, \cdots, S_n$, by the relation

$$C_n^k = \binom{n+k}{k}^{-1} S_n^k,$$

where

$$S_n^0 = S_n \quad \text{and} \quad S_n^k = \sum_{v=0}^{n} S_v^{k-1} \quad \text{for } n \geq 0, \quad k = 1, 2, \cdots,$$

then it follows [1, p. 96] that

$$(1.1) \qquad S_n^{k+m} = \sum_{v=0}^{n} \binom{n-v+m-1}{m-1} S_v^k$$

and

$$(1.2) \qquad S_n^k = \sum_{v=0}^{m} (-1)^v \binom{m}{v} S_{n-v}^{k+m} \qquad (m = 1, 2, \cdots).$$

The only known analogues to these formulas for the Hölder mean that this writer has been able to find are as follows. Denoting the k^{th} order Hölder mean of the terms $S_0, S_1, \cdots, S_n$ by H_n^k, and recalling the definition that

$$H_n^0 = S_n \quad \text{and} \quad H_n^k = \frac{1}{n+1} \sum_{v=0}^{n} H_v^{k-1} \qquad \text{for } n \geq 0, \quad k = 1, 2, \cdots,$$

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