SOME EXTENSION THEOREMS FOR CONTINUOUS FUNCTIONS

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1. Introduction. In a recent paper, J. Dugundji proved [**11**, Th. 4.1] that every convex subset Y of a locally convex topological linear space has the following property:

(1) If X is a metric space, A a closed subset of X, and f a continuous function from A into Y, then f can be extended to a continuous function from X into Y.

Let us call a topological space Y which has property (1) an *absolute extensor for metric spaces,* and let *absolute extensor for normal (or paracompact, etc.) spaces* be defined analogously. According to Dugundji's theorem above, the supply of spaces which are absolute extensors for metric spaces is quite substantial, and it becomes reasonable to ask the following question:

(2) Suppose that Y is an absolute extensor for metric spaces. Under what conditions is it also an absolute extensor for normal (or paracompact, etc.) spaces?

Most of this paper (§§2-6) will be devoted to answering this question and related questions. The related questions arise in connection with the concepts of absolute retract, absolute neighborhood retract, and absolute neighborhood extensor (in §2 these are all defined and their interrelations and significance explained), and it is both convenient and natural to answer all the questions simultaneously. Assuming that the space Y of (2) is metrizable, we are able to answer these questions completely (thereby solving some heretofore unsolved problems of Arens [**2**, p. 19] and Hu [**18**]) in Theorems 3.1 and 3.2 of §3; §§4 and 5 are devoted to proving these theorems. In §6 we show by an example that things can go completely awry if Y is not assumed to be metrizable.

Our final section (§7), which is also based on Dugundji's [**11**, Th. 4.1], deals with *simultaneous* extensions of continuous functions. It is entirely independent of §§2-6, and is the only part of this paper which might interest those readers who are interested only in metric spaces.

We conclude this introduction with a summary of some of the less familiar

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