ON THE PRIME IDEALS OF THE RING OF ENTIRE FUNCTIONS

MELVIN HENRIKSEN

1. Introduction. Let R be the ring of entire functions, and let K be the complex field. In an earlier paper [6], the author investigated the ideal structure of R, particular attention being paid to the maximal ideals. In 1946, Schilling [9, Lemma 5] stated that every prime ideal of R is maximal. Recently, I. Kaplansky pointed out to the author (in conversation) that this statement is false, and constructed a nonmaximal prime ideal of R (see Theorem 1(a), below). The purpose of the present paper is to investigate these nonmaximal prime ideals and their residue class fields. The author is indebted to Prof. Kaplansky for making this investigation possible.

The nonmaximal prime ideals are characterized within the class of prime ideals, and it is shown that each prime ideal is contained in a unique maximal ideal. The intersection P^* of all powers of a maximal free ideal M is the largest nonmaximal prime ideal contained in M. The set P_M of all prime ideals contained in M is linearly ordered under set inclusion, and distinct elements P of P_M correspond in a natural way to distinct rates of growth of the multiplicities of the zeros of functions f in P.

It is shown that the residue class ring R/P of a nonmaximal prime ideal P of R is a valuation ring whose unique maximal ideal is principal; R/P is Noetherian if and only if $P = P^*$. The residue class ring R/P^* is isomorphic to the ring $K\{z\}$ of all formal power series over K. The structure theory of Cohen [2] of complete local rings is used.

2. Notation and preliminaries. A familiarity with the contents of [6] is assumed, but some of it will be reproduced below for the sake of completeness.

DEFINITION 1. If $f \in R$, and I is any nonvoid subset of R, let:

(a) $A(f) = [z \in K \mid f(z) = 0]$ (Note that multiple zeros are repeated. Unions and intersections are taken in the same sense.);

(b) $A(I) = [A(f) \mid f \in I]$;

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