ON SUMS OF SERIES OF COMPLEX NUMBERS

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1. Introduction. We recall certain facts about the convergence of series.

1.1. Let $\sum_{i=1}^{\infty} a_i$ be a series of real numbers, $a_i \longrightarrow 0$. Then it is obvious that a sequence of signs $\epsilon_i = \pm 1$ $(i = 1, 2, \cdots)$ may be chosen so that $\sum_{i=1}^{\infty} \epsilon_i a_i$ is convergent. It is, furthermore, well known that all the possible sums so obtained form a perfect set, and if $\sum_{i=1}^{\infty} |a_i| = \infty$ then any preassigned sum may be obtained.

1.2. The first statement remains true also for complex numbers. Aryeh Dvoretzky and the author [2] proved that if $\sum_{i=1}^{\infty} c_i$ is a series of complex numbers with $c_i \longrightarrow 0$, then a sequence of signs $\epsilon_i = \pm 1$ $(i = 1, 2, \cdots)$ may be chosen so that $\sum_{i=1}^{\infty} \epsilon_i c_i$ converges and

$$\left| \sum_{i=1}^{n} \epsilon_i c_i \right| \leq \sqrt{3} \cdot \max |c_i| \qquad (n = 1, 2, \cdots).$$

1.3. The object of the present paper is to determine the sets of points which may be sums of the series $\sum_{i=1}^{\infty} \epsilon_i c_i$ when suitable sequences ϵ_i are chosen.

2. Notation and definitions. In this paper the following notations and definitions will be used.

2.1. NOTATION.

$c = a + ib$ denotes a term of a (finite or infinite) series of complex numbers, a being its real and ib its imaginary part;

$C = A + iB$ also denotes a complex number;

$\gamma = \alpha + i\beta$ denotes a direction in the plane of complex numbers, and also a unit vector in the same direction;

(C, C') is the scalar product of the vectors C and C'; that is $(C, C') = AA' + BB'$;

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