CHANGES OF SIGN OF SUMS OF RANDOM VARIABLES

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1. Introduction. Let $x_1, x_2, \cdots$ be independent random variables all having the same continuous symmetric distribution, and let

$$s_k = x_1 + \cdots + x_k .$$

Our purpose is to prove statements concerning the changes of sign in the sequence of partial sums $s_1, s_2, \cdots$ which do not depend on the particular distribution the x_k may have.

The first theorem estimates the expectation of N_n, the number of changes of sign in the finite sequence $s_1, \cdots, s_{n+1}$. Here and later we write $\phi(k)$ for

$$\frac{2([k/2]+1)}{k+1} \binom{k}{[k/2]} 2^{-k} \approx (2\pi k)^{-1/2}.$$

THEOREM 1.

$$\sum_{k=1}^{n} \frac{1}{2(k+1)} \leq \mathrm{E}\{N_n\} \leq \frac{1}{2} \sum_{k=1}^{n} \phi(k).$$

It is known (see [**1**]) that, with probability one,

$$(1) \qquad \limsup_{n \to \infty} \frac{N_n}{(n \log \log n)^{1/2}} = 1$$

when the x_k are the Rademacher functions. We conjecture, but have not been able to prove, that (1) remains true, provided the equality sign be changed to $\leq$, for all sequences of identically distributed independent symmetric random variables. We have had more success with lower limits:

THEOREM 2. *With probability one,*

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