MAPPING PROPERTIES OF CESÀRO SUMS OF ORDER TWO OF THE GEOMETRIC SERIES

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1. Introduction. Previous investigations of the mappings

$$w = S_n^{(k)}(z)$$

of the unit circle $|z| \leq 1$, where

$$S_n^{(k)}(z) = \binom{n+k}{k} + \binom{n+k-1}{k} z + \cdots + \binom{k}{k} z^n$$

denotes the nth Cesàro sum of order k of the geometric series, have been made by Fejér, Schweitzer, Sidon, and Szegö. Knowledge of the properties of the sums $S_n^{(k)}(z)$ is valuable in the study of power series having coefficients monotonic of order $k+1$.

The present article provides additional asymptotic properties for

$$S_n^{(2)}(e^{i\phi}) = x_n(\phi) + i y_n(\phi).$$

The following results are established:

THEOREM 1. *For n sufficiently large, an* α_n *exists such that* $y_n(\phi)$ *is increasing for* $0 < \phi < \alpha_n$ *and decreasing for* $\alpha_n < \phi < \pi$. *Furthermore,*

$$\alpha_n = \alpha/n + O(n^{-2}), \quad \text{where } \pi < \alpha < 3\pi/2.$$

THEOREM 2. *For n sufficiently large, a* β_n *exists such that*

$$x_n'(\phi) \begin{cases} \leq 0, \ 0 < \phi < \beta_n, \ n \equiv 0 \pmod 3 \\ < 0, \ 0 < \phi < \beta_n, \ n \equiv 1 \pmod 3 \\ < 0, \ 0 < \phi < \beta_n, \ n \equiv 2 \pmod 3 \end{cases}$$

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