THE TWO NONCHARACTERISTIC PROBLEM WITH DATA PARTLY ON THE PARABOLIC LINE

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1. Introduction. We consider the equation

$$K(y)u_{xx} + u_{yy} = 0, \tag{1}$$

where $K(y)$ is a monotone increasing, twice differentiable function of y with $K(0) = 0$. The equation is elliptic for $y > 0$, hyperbolic for $y < 0$, and $y = 0$ is a parabolic line. Equations of this type have been of interest recently because of certain problems arising in transonic flow. The equations for the compressible flow of an ideal fluid when transformed to the hodograph plane lead, in the transonic case, to an elliptic-hyperbolic equation of the above type.

In this paper the existence and uniqueness of the solution of a certain boundary value problem are discussed. It will be clear from the methods employed that estimates can be obtained for the solution in terms of the boundary values, although these estimates are not stated explicitly.

Equation (1) has real characteristics in the lower half-plane given by the equations

$$\frac{dy}{dx} = + \frac{1}{\sqrt{-K}}, \tag{2a}$$

$$\frac{dy}{dx} = - \frac{1}{\sqrt{-K}}. \tag{2b}$$

Let γ_1 be the characteristic of (2b) passing through $(0, 0)$, and γ_2 the member of (2a) passing through $(2, 0)$. Then the segment $0 \le x \le 2$, along with γ_1 and γ_2, will enclose a domain which we denote by D'. Let Γ, given by $y = h(x)$, be a curve lying in D' and emanating from the point $(2, 0)$. It will be assumed that $h(x)$ intersects each characteristic of (1) at most once, and that there are two positive constants m and M such that $0 < m \le h'(x) \le M$. We call

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