SOME PERTURBED ELECTROSTATIC FIELDS

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Summary. When a dielectric body is placed in an electrostatic field, the field becomes perturbed, and the boundary of the dielectric experiences a mechanical force due to the refraction of the lines of force. This paper is concerned with finding expressions for the resultant mechanical force at a surface of discontinuity separating two media of different specific inductive capacities, and also with determining the perturbed fields in both dielectrics (assumed homogeneous and isotropic) for various boundary shapes. In the first section, some two-dimensional fields are discussed, starting with a concentric circular cylindrical system, and then proceeding to other cases. The second section is devoted to an application of Stokes' stream function to three-dimensional axisymmetric fields.

I. TWO-DIMENSIONAL FIELDS

1.1. Mechanical force and couple. Consider a nested system of homogeneous isotropic dielectric cylinders, and let the perturbed electrostatic fields in the various media be expressed in terms of complex potential functions. Taking the origin of coordinates inside the contour of the interface separating the medium (s) of dielectric constant k_s from the medium $(s+1)$ of dielectric constant k_{s+1}, we know that the interface C will experience a force whose components are given by [4]

$$(1) \qquad Y + iX = \frac{k_{s+1}}{8\pi} \int_C \left(\frac{dw_{s+1}}{dz}\right)^2 dz - \frac{k_s}{8\pi} \int_C \left(\frac{dw_s}{dz}\right)^2 dz,$$

where w_{s+1} is the complex potential in the medium $(s+1)$, and w_s that in the medium (s).

The anticlockwise moment about the origin of the mechanical forces is

$$(2) \qquad \Gamma = \Re \frac{k_{s+1}}{8\pi} \int_C \left(\frac{dw_{s+1}}{dz}\right)^2 z\,dz - \Re \frac{k_s}{8\pi} \int_C \left(\frac{dw_s}{dz}\right)^2 z\,dz.$$

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