METHODS OF SUMMATION

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1. Methods of Rogosinski and Bernstein. In this note we shall discuss certain matrix methods of summation, though otherwise § 1 and § 2 are unrelated. In this section we wish to consider some of the properties of the method (B^h), where we say that a series $\sum_{\nu=0}^{\infty} u_\nu$ is summable (B^h) when

$$B_n^h = \sum_{\nu=0}^{n} u_\nu \cos \frac{\pi}{2} \left(\frac{\nu}{n+h}\right) \longrightarrow S, \; n \longrightarrow \infty.$$

The method (B^h) has been the subject of recent papers by Agnew [1], Karamata [5, 6], and Petersen [7]. It has been shown in the papers by Agnew and Petersen that for $h > 1/2$ the method (B^h) is equivalent to the arithmetic means of Cesaro (C), and in the paper by Agnew that for $0 < h < 1/2$ the method is equivalent to methods stronger than (C).

We shall now construct examples after a method of Hurwitz [4], to show that for $h < 0$ the method (B^h) sums a series not summable (C). Hence, since all series summable (C) are summable (B^h), we shall have proved that (B^h) is stronger than (C).

We shall first consider $-1 < h < 0$, so that all the coefficients in any row are positive except the nth coefficient $\cos \{\pi n/[2(n+h)]\}$. We choose $u_0 > 1$ and assume that the first $m-1$ terms of the series $\sum_{\nu=0}^{\infty} u_\nu$ are known. Then we select u_m so that

$$B_m^h = \sum_{\nu=0}^{m} u_\nu \cos \frac{\pi}{2} \left(\frac{\nu}{m+h}\right) = 0,$$

or

$$-u_m \cos \frac{\pi}{2} \left(\frac{m}{m+h}\right) = \sum_{\nu=0}^{m-1} u_\nu \cos \frac{\pi}{2} \left(\frac{\nu}{m+h}\right).$$

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