APPLICATION OF A THEOREM OF PÓLYA TO THE SOLUTION OF AN INFINITE MATRIX EQUATION

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1. Introduction. G. Pólya [2] has given various sufficient conditions on the infinite matrix A to ensure that the infinite system of linear equations $Au = b$, where b and u are column vectors, has a solution in u. It is remarkable that there are no conditions on the given column vector b.

R. G. Cooke [1, pp. 34-35] established the existence of reciprocals of a matrix A satisfying Pólya's conditions, given in the following theorem.

THEOREM 1 (Pólya). *In the infinite system of linear equations*

$$(1.1) \qquad \sum_{j=1}^{\infty} a_{i,j}\, u_j = b_i \qquad (i = 1, 2, 3, \cdots),$$

where $\{b_i\}$ *is an arbitrary sequence, let* $(a_{i,j})$ *satisfy the conditions*

(i) *the first row* $a_{1,j}$ *contains an infinity of nonzero elements, and*

(ii) $\liminf_{j\to\infty} \dfrac{|a_{1j}| + |a_{2j}| + \cdots + |a_{i-1,j}|}{|a_{i,j}|} = 0$ *for every fixed* $i \geq 2$.

Then there exists an infinite sequence $\{u_j\}$ *satisfying* (1.1), *such that all the left sides are absolutely convergent.*

It follows [1, pp. 34-35] that if a matrix $A = (a_{i,j})$ satisfies (i) and (ii), then A has an infinity of linearly independent right-hand reciprocals, and that if A', the transpose of A, satisfies (i) and (ii), then A has an infinity of linearly independent left-hand reciprocals.

In this paper it is shown that Pólya's theorem can be applied to establish the existence of solutions of the infinite matrix equation

Received December 8, 1952, and in revised form June 15, 1953. The author wishes to express his thanks to Dr. R. G. Cooke and Dr. P. Vermes, who have read the manuscript and made some useful comments. He also thanks the referee for his comments on Theoems 2 and 4, which were of assistance in clarifying the proofs.

Pacific J. Math. 4 (1954), 21-28