ON TWO PROBLEMS OF KUREPA

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We prove [1]:

THEOREM 1. *There exists a denumerable ramified partially ordered set with the property that there is no chain meeting all maximal anti-chains and no anti-chain meeting all maximal chains.*

(Here a *chain* (*anti-chain*) is a set of elements every pair of which are comparable (incomparable). A *ramified* partially ordered set S is one in which for each x in S the set of elements $\leq x$ forms a chain.)

Proof. We denote by F the set of all finite sequences $(\alpha_1, \alpha_2, \cdots, \alpha_k)$ of integers. We use Greek letters α, β to denote elements of F, we denote by $l(\alpha)$ the length of α (that is, the number of terms in the sequence α) and by α_i (for $i = 1, \cdots, l(\alpha)$) the ith term in the sequence α; i, k are used throughout as variables for positive integers. If n is an integer we denote by (α, n) the sequence $(\alpha_1, \cdots, \alpha_{l(\alpha)}, n)$ obtained by adding the term n to the sequence α. We define $\alpha \leq \beta$ to hold when conditions

$$A: l(\alpha) \leq l(\beta),$$

$$B: \alpha_i = \beta_i \quad \text{for } i = 1, \cdots, l(\alpha) - 1,$$

and

$$C: \alpha_{l(\alpha)} \leq \beta_{l(\alpha)},$$

are all satisfied. It is easily seen that this relation '$\leq$' is a ramified partial ordering of F.

Now let L_α denote the chain of elements $\leq \alpha$, let C_α denote the set of

[1] This answers two questions posed by Kurepa (Pacific J. Math. 2 (1952), 323-326). Answers to these questions were found independently by W. Gustin; see the reviews in Math. Rev. 14 (March, 1953), p. 255 by W. Gustin, and in Zentralblatt für Math., 64 (1953), p. 52, by J. C. Shepherdson.

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