ON A THEOREM OF BEURLING AND KAPLANSKY

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1. Introduction. The object of this paper is to remark that a natural and simple proof of the theorem of Beurling and Kaplansky (Theorem 1 below) can be obtained by adapting to general groups a classical proof already given in the books of Wiener [8] and Zygmund [9]. In fact, Theorem 1 is an immediate consequence of a lemma (Lemma 1 below) which was proved by these authors in the case when the group is the integers or the real numbers. An easy generalization of Lemma 1 (Lemma 2 below) yields immediately the generalization of the Beurling and Kaplansky theorem stated as Theorem 2 below. For the history of the development of this theorem, see [3, p. 149] and [5]; the book [3] did not appear until the present paper had been submitted, but it seemed wise to add the reference.

2. Statement of results. Let $A = \{a, b, \cdots\}$ be a locally compact abelian group and $X = \{x, y, \cdots\}$ the dual group (the group operations will be written multiplicatively). Let

$$L^1(A) = \{f, g, h, p, \cdots\}$$

denote the set of all integrable functions with respect to the Haar measure of A,

$$||f|| = ||f||_1$$

the L^1-norm of f, $\hat{f}(x)$ the Fourier transform of $f(a)$,

$$f_1 * f_2$$

the product of convolution (that is, the product in the group algebra),

$$f_1 f_2 = f_1(a)\, f_2(a)$$

the ordinary product of functions, and

$$(x, a) = x(a) = a(x)$$

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