SPECTRAL OPERATORS

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1. Introduction. The present paper and the five following it by S. Kakutani, J. Wermer, W. G. Bade, and J. Schwartz are all related; in them we discuss different aspects of the problem of the complete reduction of an operator. A spectral operator is a linear operator on a complex Banach space which has a resolution of the identity.[1] It is shown that a bounded operator T is spectral if and only if it has a canonical decomposition of the form

$$T = S + N,$$

where S is a scalar type operator and N is a generalized nilpotent commuting with S. By a scalar type operator is meant a spectral operator S with resolution of the identity E which satisfies the equation

$$S = \int_{\sigma(S)} \lambda E(d\lambda).$$

The scalar part S of T and the radical part N of T are uniquely determined by T. For analytic functions f one has an operational calculus given by the formula

$$f(T) = \sum_{n=0}^{\infty} \frac{N^n}{n!} \int_{\sigma(T)} f^{(n)}(\lambda) E(d\lambda).$$

Some spectral operators are of type m; that is, the above formula reduces to

$$f(T) = \sum_{n=0}^{m} \frac{N^n}{n!} \int_{\sigma(T)} f^{(n)}(\lambda) E(d\lambda),$$

and in Hilbert space conditions on the resolvent are given which are equivalent to the statement that the spectral operator T is of type m. Spectral operators T

[1] Formal definitions will be given later.

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