TRANSFORMATIONS OF SYSTEMS OF RELATIVISTIC PARTICLE MECHANICS

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1. Introduction. In [7] the axiomatic foundations of classical particle mechanics were investigated; and in [8] the transformations which carry systems of classical particle mechanics into systems of classical particle mechanics were determined. The purpose of the present paper is a similar investigation of relativistic particle mechanics (in the sense of the special theory of relativity). Some remarks on the general orientation of these studies are to be found in [7, § 1] and in [9].

In regard to our axiomatization of relativisitic particle mechanics, we want to emphasize that we have in no sense attempted to use primitive notions which are logically or epistemologically simple. Investigations with these latter aims are to be found in [11], [12], [13], and [14]; but these studies are incomplete in the sense that they do not give axioms adequate for relativisitic particle mechanics as it is ordinarily conceived by physicists. We have attempted to present such a complete set of axioms in a mathematically clear way.

The main result of the present paper is the determination under a certain weak hypothesis of the set of transformations which always carry systems of relativistic particle mechanics into systems of relativistic particle mechanics. Although this set of transformations is not a group (under the usual operation) we are able to show that it is essentially a Brandt groupoid. It is difficult precisely to compare our results with those in [6], but our results seem to represent an improvement in three respects: (i) we work within an explicit axiomatic framework; (ii) we consider transformations of the units of mass and force as well as position and time; (iii) we consider transformations from one value for the velocity of light to another.

We briefly summarize the mathematical notations we use, most of which are standard. We denote the ordered n-tuple whose first member is a_1, whose second member is a_2, and so on, by

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