LINEAR FUNCTIONAL EQUATIONS AND INTERPOLATION SERIES

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1. Introduction. The question of obtaining complete sets of solutions for a given linear partial differential equation is of the greatest interest from the theoretical as well as from the computational point of view. For constructing such sets, several methods of considerable generality have been proposed. Thus, for instance, Bergman [3] has introduced an integral operator which provides a means for the generation of complete sets when the differential equation is of the second or the fourth order. Extensions may be made to higher orders. By means of Bergman's operator, the space of analytic functions of a single complex variable is mapped upon the space of solutions of the given differential equation, and the process yields a generalization of the operator Re in the case of harmonic functions.

Complete sets of solutions may also be found by a method which is analogous to Runge's method of approximation in the theory of analytic functions. A description of this may be found in [6, p. 282]. This scheme has the practical drawback of requiring a knowledge of a fundamental singularity for the differential equation, a function which is known explicitly for but few differential equations.

In the present paper, we adopt a different point of view and study possible representations of solutions of linear functional equations of a certain class, and the generation of complete sets of such solutions by means of generalized interpolation series. By this is meant a biorthogonal series of the form

$$(1) \qquad f \sim \sum_{n=0}^{\infty} L_n(f)\phi_n; \quad L_m(\phi_n) = \delta_{mn} .$$

Here $\{L_n\}$ is a sequence of linear functionals. When each L_n is a point or a linear differential operator, then the series (1) reduces to a classical interpolation series. Our method is, essentially, to reduce the problem of the solution of the linear functional equation to a problem involving a denumerable infinity

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