A NONLINEAR BOUNDARY VALUE PROBLEM FOR SECOND ORDER DIFFERENTIAL SYSTEMS

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1. Introduction. Studies of nonlinear differential systems have become increasingly important with recent advances in all areas of applied mathematics. Linear systems, and methods based upon these, are inadequate to describe and investigate many of the phenomena associated with physical, chemical, and other systems. In many cases, linearizing processes applied to the equations impose properties of existence, uniqueness, oscillation character, or other nature, which effectively eliminate the phenomenon of prime concern from the investigation. New methods for studying nonlinear systems which avoid restrictions of the type just suggested are needed and are in process of development.

The present paper is concerned with a second-order nonlinear ordinary differential system in the real domain, with which are associated linear boundary conditions at two points. The methods used, and the results obtained, generalize and extend those given in an earlier paper [3]. In particular, the boundary conditions treated in the present paper fail to be self-adjoint when they are associated with linear differential equations. The paper establishes the existence of sets of characteristic numbers (eigenvalues) for the nonlinear systems, and gives oscillation theorems for the associated solutions.

2. Results. In the differential system,

$$(1)\qquad \begin{aligned} dy/dx &= K(x, y, z; \lambda)\,z\,, \\ dz/dx &= G(x, y, z; \lambda)\,y\,, \end{aligned}$$

let $K(x, y, z; \lambda)$, $-G(x, y, z; \lambda)$ be real positive functions that are continuous in $(y, z; \lambda)$ on

$$SL\begin{cases} S: -\infty < y, z < +\infty, \\ L: L_1 < \lambda < L_2\,, \end{cases}$$

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