THE BEHAVIOR OF SOLUTIONS OF A LINEAR DIFFERENTIAL EQUATION OF SECOND ORDER

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Introduction. This paper is a study of the oscillation and boundedness of solutions of the self-adjoint differential equation

$$(r(x)y')' + p(x)y = 0 \tag{1}$$

on the infinite half-axis I, $a \leq x < +\infty$. We shall assume throughout that $r(x)$ and $p(x)$ are real, continuous functions and that $r(x)$ is positive on I. A non-null solution of equation (1) is said to be oscillatory if it has an infinity of zeros on I.

It will be noted that the results given here are of the "integral test" variety. Although the problem goes back at least to Kneser [**5, 6**]; probably the first "integral" condition for oscillation is due to Fite [**1**]. His criterion is that *all solutions of the even order equation*

$$y^{(2n)} + p(x)y = 0 \tag{2}$$

oscillate provided $p(x) > 0$ *and*

$$\int_a^\infty p(x)\,dx = +\infty.$$

A similar result for the case $n = 1$ is due to Wintner [**14**] in which there is no restriction on the sign of $p(x)$. Simultaneously Leighton [**8**] noted the analogous result for equation (1) (see Theorem 1 in this paper).

Hille [**2**] studied the nonoscillation of solutions of (1) for the case $r(x) \equiv 1$ and $p(x)$ nonnegative and established the roles of the functions

$$\int_a^x \xi p(\xi)\,d\xi \quad \text{and} \quad x\int_x^\infty p(\xi)\,d\xi$$

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