A MODIFIED SCHNIRELMANN DENSITY

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1. Introduction. We define a modified Schnirelmann density for an infinite sequence of positive integers and prove two theorems about this density which are analogous but not identical to well-known results for Schnirelmann density.

Henceforth we assume all sequences are infinite. Let A be a sequence of positive integers $a_1 < a_2 < \cdots$, let $A(n)$ be the number of integers of A not greater than n, and let I be the sequence of all positive integers. Then the well-known asymptotic density $\delta(A)$, the Schnirelmann density α, and the modified Besicovitch density α_1, of A, are defined as follows:

$$\delta(A) = \liminf \frac{A(n)}{n};$$

$$\alpha = \operatorname{glb} \frac{A(n)}{n};$$

$$\alpha_1 = \operatorname*{glb}_{n \geq s} \frac{A(n)}{n+1},$$

where $A \neq I$ and s is the smallest positive integer missing from A. We define the modified Schnirelmann density or more briefly the modified density α^* of A as follows:

$$\alpha^* = \operatorname{glb} \frac{i}{a_i}.$$

Thus the modified density may be defined by merely restricting to A the n occurring in the definition of Schnirelmann density.

Let B be the sequence of positive integers $b_1 < b_2 < \cdots$. The sum $C = A + B$ of the sequences A and B is defined as the sequence of integers of the form

Received May 29, 1953. Results in this paper were included in a doctoral dissertation written under the direction of Professor Ivan Niven at the University of Oregon, 1953.

Pacific J. Math. 5 (1955), 119-124