ON THE NUMBER OF PRIMITIVE PYTHAGOREAN TRIANGLES WITH AREA LESS THAN n

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1. Introduction. In the preceding paper Lambek and Moser have shown that if $P_a(n)$ is the number of primitive Pythagorean triangles with area less than n then

$$P_a(n) = cn^{1/2} + O(n^{1/3}), \tag{1}$$

where

$$c = \frac{\Gamma^2(1/4)}{2^{1/2}\pi^{5/2}}.$$

They conjecture that

$$P_a(n) = c\,n^{1/2} - c'n^{1/3} + o(n^{1/3}), \tag{2}$$

and on the basis of a table due to Miksa they find

$$c' \approx .295. \tag{3}$$

Our purpose here is to show that

$$P_a(n) = c\,n^{1/2} - c'n^{1/3} + O(n^{1/4}\ln n), \tag{4}$$

where

$$c' = -\frac{\zeta(1/3)(1+2^{-1/3})}{\zeta(4/3)(1+4^{-1/3})} \approx .297. \tag{5}$$

In the paper by Lambek and Moser, the problem of calculating $P_a(n)$ has been reduced to that of counting the number of lattice points $L(n)$ in the region R_1 defined by

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