ON THE DISTRIBUTION OF PYTHAGOREAN TRIANGLES

J. LAMBEK AND L. MOSER

1. Introduction. This paper was conceived with the object of estimating the number $P_a(n)$ of primitive Pythagorean triangles with *area* less than n. The problem seemed of interest, since F.L. Miksa [5] recently tabulated all primitive Pythagorean triangles with area less than 10^9, in order of increasing area. The method employed here also yields known estimates for the numbers $P_h(n)$ and $P_p(n)$ of primitive Pythagorean triangles with *hypotenuse* and *perimeter*, respectively, less than n; we use $P(n)$ as generic notation for all of these.

D. N. Lehmer [4] had shown in 1900 that

$$P_h(n) \sim \frac{1}{2}\pi^{-1}n, \quad P_p(n) \sim \log 2 \cdot \pi^{-2}n.$$

In 1948, D. H. Lehmer [3] obtained

$$P_p(n) = \log 2 \cdot \pi^{-2}n + O(n^{1/2}\log n),$$

pointing out that this disproved a conjecture of Krishnaswami [2] that $P_p(n) \sim n/7$. For primitive Pythagorean triangles with area less than 2.10^6, W. P. Whitlock [6] found that

$$|P_a(n) - \frac{1}{2}n^{1/2} + 5| \leq 2.$$

However, Miksa's table, which goes 500 times as far as Whitlock's, suggested that $P_a(n)$ is not asymptotic to $(1/2)n^{1/2}$.

In § 2 we reduce the problem of approximating $P(n)$ to that of estimating the number of lattice points in certain regions of the Cartesian plane. The latter problem is treated in § 3, with some attempt at generality. In § 4 we obtain the following asymptotic formulae for $P(n)$:

$$P_h(n) = \frac{1}{2}\pi^{-1}n + O(n^{1/2}\log n),$$

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