GRADIENT METHODS OF MAXIMIZATION

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1. Introduction. We shall consider the computational problem of finding a point

$$c = (c_1, c_2, \cdots, c_n)$$

for which a given function of n variables

$$f(x) = f(x_1, x_2, \cdots, x_n)$$

attains its maximum. Frequently, the form of the function and the number of independent variables involved make it prohibitively difficult to determine the point c by direct methods, and methods of successive approximations are accordingly used.

Suppose that

$$x^{(0)} = (x_1^{(0)}, x_2^{(0)}, \cdots, x_n^{(0)})$$

is an approximation to the point c. Assuming that $x^{(0)}$ is sufficiently close to c, we may obtain an improved approximation to c by considering the first few terms of the Taylor expansion of $f(x)$ about $x^{(0)}$. Presumably, the greater the number of terms of the expansion that we consider, the better will be our improved approximation and the more rapidly will the corresponding iterative procedure converge. On the other hand, increasing the number of terms of the expansion involves the calculation of higher order derivatives and increases considerably the computational cost of each iteration.

The methods of successive approximations to be discussed in this paper are (1) gradient methods using the first order derivatives only, and (2) the Newton method which uses first and second order derivatives. *In both cases, it is possible to obtain, from the successive approximations, certain relevant information about terms of order higher than those actually computed, and to conveniently use this information to improve the rate of convergence.*

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