A CLASS OF GENERALIZED WALSH FUNCTIONS

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1. Introduction. Let α denote a fixed integer, $\alpha \geq 2$, and put $\omega = \exp(2\pi i/\alpha)$.

DEFINITION 1. The *Rademacher functions of order* α are defined by

$$\phi_0(x) = \omega^k \text{ if } k/\alpha \leq x < (k+1)/\alpha,\ k = 0, \cdots, \alpha - 1;$$

and for $n \geq 0$

$$\phi_n(x+1) = \phi_n(x) = \phi_0(\alpha^n x).$$

DEFINITION 2. The *Walsh functions of order* α are defined by

$$\psi_0(x) = 1,$$

and if $n = a_1 \alpha^{n_1} + \cdots + a_m \alpha^{n_m}$ where $0 < a_j < \alpha$ and $n_1 > n_2 > \cdots > n_m$, then

$$\psi_n(x) = \phi_{n_1}^{a_1}(x) \cdots \phi_{n_m}^{a_m}(x).$$

For convenience we let Ψ_α denote the set of Walsh functions of order α. We may observe that Ψ_2 is the orthonormal system of functions defined by Walsh [**4**]. R.E.A.C. Paley's proof that Ψ_2 is orthonormal and complete in $L(0,1)$ may be modified by the reader to establish the same properties for Ψ_α, $\alpha = 3, 4, \cdots$. [**3**; pp. 242-244].

It is the purpose of this paper to study Fourier expansions in the sets Ψ_α. The results obtained here will include known results for ordinary Walsh Fourier series, most of which are contained in a paper of N. J. Fine [**1**]. In fact, most

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