SOME DETERMINANTS INVOLVING BERNOULLI AND EULER NUMBERS OF HIGHER ORDER

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1. Introduction. In this paper we evaluate certain determinants whose elements are the Bernoulli, Euler, and related numbers of higher order. In the notation of Nörlund [**1**, Chapter 6] these numbers may be defined as follows: the Bernoulli numbers of order n by

$$(1.1)\qquad \left(\frac{t}{e^t-1}\right)^n = \sum_{v=0}^{\infty} \frac{t^v}{v!} B_v^{(n)},$$

the related "D" numbers by

$$(1.2)\qquad \left(\frac{t}{\sin t}\right)^n \sum_{v=0}^{\infty} (-1)^v \frac{t^{2v}}{(2v)!} D_{2v}^{(n)} \qquad (D_{2v+1}^{(n)}=0),$$

the Euler numbers of order n by

$$(1.3)\qquad (\sec t)^n = \sum_{v=0}^{\infty} (-1)^v \frac{t^{2v}}{(2v)!} E_{2v}^{(n)} \qquad (E_{2v+1}^{(n)}=0),$$

and the "C" numbers by

$$(1.4)\qquad \left(\frac{2}{e^t+1}\right)^n = \sum_{v=0}^{\infty} \frac{t^v}{v!} \frac{C_v^{(n)}}{2^v}.$$

(By n we denote an arbitrary complex number. When $n=1$, we omit the upper index in writing the numbers; for example, $B_v^{(1)} = B_v$.)

We evaluate determinants such as

$$|B_i^{(x_j)}| \qquad (i, j = 0, 1, \cdots, m)$$

for the Bernoulli numbers, and similar determinants for the other numbers. The

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