NOTE ON THE MULTIPLICATION FORMULAS FOR THE JACOBI ELLIPTIC FUNCTIONS

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1. Introduction. For t an odd integer it is well known [**4**, vol. 2, p. 197] that

$$sn\, tx = \frac{sn\, x \cdot G_1^{(t)}(z)}{G_0^{(t)}(z)} \qquad (z = sn^2 x), \tag{1.1}$$

where

$$\begin{aligned} G_0^{(t)} &= 1 + a_{01} z + a_{02} z^2 + \cdots + a_{0t'} z^{t'}, \\ G_1^{(t)} &= t + a_{11} z + a_{12} z^2 + \cdots + a_{1t'} z^{t'} \qquad (t' = (t^2-1)/2), \end{aligned} \tag{1.2}$$

and the a_{ij} are polynomials in $u = k^2$ with rational integral coefficients. If we define

$$\beta_m(t) = \beta_m(t, u)$$

by means of

$$\frac{sn\, tx}{t\, sn\, x} = \sum_{m=0}^{\infty} \beta_{2m}(t) \frac{x^{2m}}{(2m)!} \qquad (\beta_{2m+1}(t) = 0), \tag{1.3}$$

it follows from (1.1) and (1.2) that $t\beta_{2m}(t)$ is a polynomial in u with integral coefficients for all m and all odd t. We shall show that

$$\beta_{2m}(t) = H_m(t) - \sum_{\substack{p-1 \mid 2m \\ p \mid t}} \frac{1}{p} A_p^{2m/(p-1)}(u), \tag{1.4}$$

where $H_m(t) = H_m(t, u)$ denotes a polynomial in u with integral coefficients,

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