GENERALIZED WALSH TRANSFORM

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Introduction. The Walsh functions were first defined by Walsh [6] as a completion of the Rademacher functions in the interval $(0,1)$. As originally defined $\psi_n(x)$ took on the values ± 1. The generalization of Chrestenson [1] permits $\psi_n(x)$ to have the values $e^{2\pi n i/\alpha}$ for some integer α, and also leads to a complete orthonormal system over $[0,1]$. Fine [2] considers the original Walsh function, but with arbitrary subscript, attained by consideration of certain dyadic groups. This paper combines these two generalizations by starting with the Walsh functions as defined by Chrestenson and then using a subsidiary result of Fine to define a Walsh function $\psi_y(x)$ for arbitrary subscript.

With $\psi_y(x)$ one can define a Walsh-Fourier transform for functions in $L_p(0, \infty)$, $1 \leq p \leq 2$. Many of the ordinary Fourier transform theorems carry over, with certain modifications. For $1 < p \leq 2$ the transform is defined as a limit in the appropriate mean, with a Plancherel theorem holding for $p = 2$. Since the proofs carry over from ordinary transforms, or from the L_1 theory only a few theorems are stated for these cases with only brief proofs. The case of L_1 requires considerably more preparation.

Section one is devoted to definitions and obtaining certain varied but very necessary results, such as the evaluation of definite integrals of $\psi_y(x)$ over specified intervals, which are used constantly throughout the paper. Walsh-Fourier series are introduced, and some of the sufficient conditions for convergence of such a series to the generating function are listed but not proved, since the proofs are available in Chrestenson's paper [1].

Section two covers certain basic results for L_1 transforms and associated kernals. A Riemann-Lebesgue theorem follows simply from results of section one, as do sufficient conditions for the convergence to $f(x)$ of the inverse transform of the transform of $f(x)$. The function $P_\beta(x \ominus y)$ is defined as

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