THE REFLECTION PRINCIPLE FOR POLYHARMONIC FUNCTIONS

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1. Introduction. In this paper the reflection principle for harmonic functions is extended to the more general class of p-harmonic functions. The case $p = 2$ has already been treated by R. J. Duffin [1] whose method of proof will be used in part. The formula for the reflection of a biharmonic function at a straight line segment in the plane was even previously known to H. Poritsky [3]; but he did not indicate under which conditions such a continuation would have to exist.

A function $w(x_1, x_2, \cdots, x_n)$ is called p-harmonic in a region D of the n-dimensional space, if it is of class C^{2p} and satisfies the differential equation $\Delta^p w = 0$. We shall make use of the following well-known properties:

(I) w is analytic throughout D.

(II) The following representation always exists:

$$w = \sum_{\nu=0}^{p-1} x_1^{\nu}\, u_\nu(x_1, x_2, \cdots, x_n),$$

the functions $u_1, u_2, \cdots, u_{p-1}$ being harmonic in D; conversely such a sum is always p-harmonic. As a consequence, the following decompositions are also possible

$$w = f(x_1, x_2, \cdots, x_n) + x_1^{p-k}\, g(x_1, x_2, \cdots, x_n), \quad k = 1, 2, \cdots, p-1,$$

f denoting a $(p-k)$-harmonic, g being a k-harmonic function.

2. Reflection principle.

THEOREM. *Let G denote a region of the n-dimensional space, the boundary of which contains an open subset S of $x_1 = 0$. If the function $w(x_1, x_2, \cdots, x_n)$ is p-harmonic in G and if w/x_1^{p-1} assumes the boundary value 0 on S, then w can be continued analytically across S into the reflected domain G' by putting*

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