ON FACTOR FUNCTIONS

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0. Introduction. The object of this paper is to illustrate by means of a few selected examples the application of abstract but simple methods to the study of factor functions. The methods have a considerably wider range of application then is explicitly covered here: in particular, it applies to functional transformation other than that of Fourier.

The factor problem is understood in the following sense. G will denote throughout a locally compact and abelian group; E and F will be two topological vector spaces of functions, measures or distributions on G for which the Fourier transformation is suitably defined. The transform of f is denoted generally by $\hat{f}$. A function ϕ on $\hat{G}$, the group dual to G, is said to be a (Fourier) factor of class (E, F) if and only if $\phi \cdot \hat{f}$ is the transform of some $g \in F$ each time that $f \in E$. In all cases we have in mind, E and F are each invariant under the translations by group elements $t_x (x \in G)$, and in many such cases it is convenient to subordinate the factor problem to that of finding a representation theorem for a general continuous linear mapping u of E into F which commutes with translations. The class of such mappings is denoted by $L_t(E, F)$, the notation $L(E, F)$ being reserved for the set of *all* continuous linear mappings of E into F.

The formal relationship between the two problems is expressed as follows. If ϕ is a factor of class (E, F), let u be the linear mapping of E into F which is defined by agreeing that $g = u(f)$ is to signify that $\hat{g} = \phi \cdot \hat{f}$. The continuity of u is usually a consequence of the "closed graph theorem", whilst the fact that u commutes with translations is a consequence of the way that translation effects the Fourier transformation (multiplication by characters of G). On the other hand it is not always easy to show that every $u \in L_t(E, F)$ is derivable in this manner from a factor function of class (E, F).

In most of the applications dealt with below, E and F are both Fréchet spaces. The one general property of these spaces we use is the weak relative compacity of weakly bounded subsets of the dual of such a space.

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