# THE DIRICHLET PROBLEM FOR NONLINEAR ELLIPTIC EQUATIONS

JANE CRONIN

**1. Introduction.** It is often convenient to divide the question of solving the Dirichlet problem for a nonlinear elliptic differential equation into two parts:

(A) The "local" problem, that is, the problem of solving the equation for boundary functions[1] sufficiently "close" to the boundary function of a given solution.

(B) The "extension" problem, that is, the problem of finding solutions corresponding to a given set of boundary functions if the solution for one boundary function in the set is known.

S. Bernstein takes this approach in his fundamental papers on nonlinear elliptic equations [**1**, **2**], and in other papers (See [**7**] for a bibliography.) this viewpoint is used more or less explicitly.

Problem (B) is essentially the profound and difficult problem of finding "a priori" estimates for solutions of nonlinear elliptic equations (cf. the papers by Bernstein [**1**, **2**], Schauder [**15**], Leray [**10**, **11**], and Nirenberg [**13**]. Nirenberg gives, besides important new results, a clear account of previous work in the subject.) We shall be concerned here only with problem (A), which is much simpler.

In solving (A), the usual procedure is to invoke an assumption which implies uniqueness of solution, that is, an assumption which implies that the corresponding Jacobi equation has only the zero solution. This assumption is used to prove that there is a solution for each boundary function which is sufficiently "close" to the boundary function of the given solution. If the uniqueness hypothesis is relinquished, it may turn out that the equation has several solutions or no solutions at all for some boundary function which neighbors the boundary function of the given solution [**4**, **12**]. But this is a statement about *real* solutions of the differential equation. We shall show here that if *complex*

[1] We shall refer to a set of boundary values as a boundary function.

Received October 11, 1953.

*Pacific J. Math.* 5 (1955), 335-344